On the Performance of Partitioned-Spreading CDMA with Multistage Demodulation

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Abstract— **The performance of partition-spreading CDMA (PS-CDMA), a recently proposed multiple-access system, is investigated. It is shown via variance evolution that the asymptotic channel throughput of PS-CDMA improves monotonically as the number of partitions increases. The fundamental limit of the PS-CDMA system is calculated by evaluating the spectral efficiency of the layered channel that is generated by multistage PS-CDMA demodulation. Capacity-approaching codes of high rate are suggested in the encoding for each PS-CDMA user, which justifies a low-complexity two-stage decoding scheme for PS-CDMA.**

I. Introduction

In code-division multiple-access (CDMA) systems, different transmissions are distinguished from each other by the user-specific spreading sequences. The random access nature of the wireless network introduces cross-correlations between users, which can be addressed by multiuser detection [18], [19], at the cost of increasing the complexity at the receiver.

Recently, a new technique called partioned-spreading CDMA (PS-CDMA) [1], [2] has been proposed. PS-CDMA partitions the original CDMA spread symbol into a number of sections, which are then transmitted in random order. A simple receiver is used to combine the statistics of different partitions to form reliable soft estimates of the transmitted symbol in multiple stages. This general access method has as special case the interleave-division multiple-access (IDMA) that was independently studied by Li et. al. [10], [11], [12], [13], where the number of partitions $M = N$, the spreading gain. Simulations both on BER performance [10], [13] and system coverage [1] have shown that PS-CDMA outperforms CDMA in various scenarios.

In this paper, we analyze the limit of PS-CDMA systems using variance evolution. We show that the performance of PS-CDMA monotonically improves as the number of partitions M increases. The analysis is conducted in a way that is independent of the receive power levels, hence the conclusions apply to arbitrary user-power profile. The quantitative investigation of the evolving variance values shows that PS-CDMA with a few number of partitions converges very fast to that of IDMA, under quite general conditions. Viewing PS-CDMA as a nonlinear signal separator, the underlying layered channel throughput is shown to approach the channel capacity for a wide range of SNRs.

The rest of the paper is organized as follows. Section II introduces PS-CDMA and a simplified model for the multistage demodulation. In Section III-A, we develop two variance evolution (VE) functions to monitor the dynamics of the system. In Section III-B, the variance evolution analysis is developed. The performance of PS-CDMA systems encoded by non-trivial FEC codes and decoded with an iterative receiver is discussed in Section III-C. In Section III-E, the pros and cons of implementing an PS-CDMA system are presented. The underlying channel throughput is derived in Section IV, which shows a performance approaching channel capacity over a wide range of SNRs. The conclusions are summarized in Section V.

II. PS-CDMA System Model

A general partitioned-spreading CDMA (PS-CDMA) communication system is proposed by Schlegel and Kempter in [1]. In PS-CDMA, the traditional CDMA spread symbols are each partitioned into M portions which are then interleaved and transmitted over the multiple-access channel. For each partition, simple interference cancellation is performed to reduce the interference from other transmissions. The M partitions belonging to one symbol bear the same information and can be deemed as the symbols of an equivalent rate-$1/M$ repetition code, which can be easily decoded following the *a posteriori* probability (APP) principle.

Figure 1 shows the multistage reception for one PS-CDMA symbol. Each multiple-access node (MND) performs soft in-

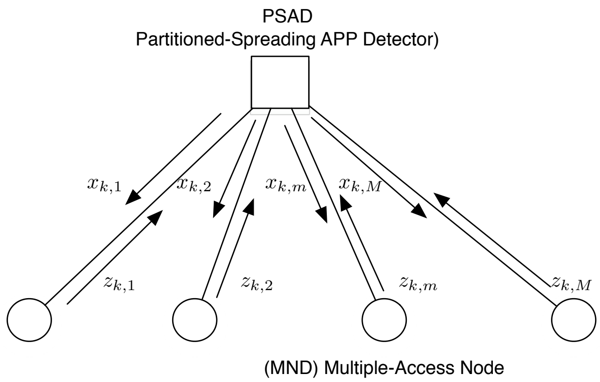

Fig. 1. Receive model for M partition-spreading CDMA

terference cancellation, generating a next-stage received signal for partition m of user k and symbol t_k

$$r_{k,m}(t_k) = \frac{\sqrt{P_k}}{M} d_k(t_k) + \sum_{j \neq k} \frac{\sqrt{P_j}}{M} \rho_{km,jl} \left(d_j(t_j) - \hat{d}_j(t_j) \right) + n_{k,m}(t_j), \quad (1)$$

where $\rho_{km,jl}$ is the cross-correlation between k-th user's m-th partition and j-th user's l-th partition, and $n_{k,m}(t_k) \sim \mathcal{N}(0, \sigma^2/M)$ is the noise sample. For both simplicity and brevity, we assume the channel is aligned in time on the partition level, hence the time index can be dropped in (1).[1] The LLR of $r_{k,m}$ can be calculated as

$$z_{k,m} = 2\frac{\mu(r_{k,m})}{\sigma^2(r_{k,m})} r_{k,m}, \quad (2)$$

which is a Gaussian consistent random variable, i.e., $x \sim \mathcal{N}(\mu_x, 2\mu_x)$. In (2), $\mu(r_{k,m}) = \sqrt{P_k}/M$, and $\sigma^2(r_{k,m})$ is the interference variance for $r_{k,m}$, given by

$$\sigma^2(r_{k,m}) \longrightarrow \frac{1}{MN} \sum_{j \neq k} P_j \sigma^2_{d,j} + \frac{\sigma^2}{M}, \quad (3)$$

where $\sigma^2_{d,j} = E\left[(d_j - \hat{d}_j)^2\right]$ is the symbol variance for user j after cancellation, and the limit in (3) is reached by taking the mild large-scale system assumption that $K, N \to \infty, M, K/N$ is fixed as in [9]. Note that the spreading gain for each sub-partition is N/M. Due to the constraint that exists between the mean and variance for a Gaussian consistent random variable, we can characterize $z_{k,m}$ by its normalized variance

$$\sigma^2_{z,k} = \frac{\sigma^2(r_{k,m})}{\mu^2(r_{k,m})} = \frac{M}{N} \sum_{j \neq k} \frac{P_j}{P_k} \sigma^2_{d,j} + \frac{M}{2\gamma_k}. \quad (4)$$

In (4), $\gamma_k = P_k/(2\sigma^2)$ is the signal-to-noise ratio for user k. We note that normalized variance is independent to the partition index m, hence we drop it in (4).

LLRs $z_{k,m}, m = 1, 2, \ldots, M$, that belong to the same symbol of user k are then combined at the PSAD node in Figure 1 using the *a posteriori* probability (APP) principle for the repetition code[2], to form the extrinsic LLR $x_{k,m}$ as

$$x_{k,m} = \sum_{l \neq m} z_{k,l}, \quad (5)$$

with the normalized variance

$$\sigma^2_{x,k} = \frac{M}{M-1} \left(\frac{1}{NP_k} \sum_{j \neq k} P_j \sigma^2_{d,j} + \frac{1}{2\gamma_k} \right). \quad (6)$$

Soft bits are generated from $x_{k,m}$ as $\hat{d}_k = \tanh(x_{k,m}/2)$, and the associated symbol error variance is

$$\begin{aligned} \sigma^2_{d,k} &= E\left[(d_k - \hat{d}_k)^2\right] = g(\sigma^2_{x,k}) \\ &= \int_{-1}^{+1} \frac{\sigma_{x,k}}{\sqrt{2\pi}} \left(\frac{1-y}{1+y} \right) \\ &\quad \exp\left\{ -\frac{\sigma^2_{x,k}}{8} \left(\log\frac{1-y}{1+y} - \frac{2}{\sigma^2_{x,k}} \right)^2 \right\} dy. \quad (7) \end{aligned}$$

III. Variance evolution analysis

A. Choice of variance evolution functions

Equation (6) shows the pro and cons of introducing the M-fold partition scheme by the scaling factor $\frac{M}{M-1}$. M in the nominator indicates that the spreading gain is reduced by a factor of M, or, in other words, the effective number of users increases by M. On the other hand, the APP processing at PSAD node reduces the effective number of users by $M-1$, which appears in the denominator. In the remainder of this paper, we answer the question if there is always an improvement as M increases and how this improvement is achieved, from the perspective of variance evolution.

Our approach is to use functions (6) and (7) to observe the dynamic behavior of the system. We note that (6) corresponds to one stage going through both MND and PSAD, while (7) represents the operation that transforms LLR $x_{k,m}$ into a soft bit $\hat{d}_k$. As a result, (7) is a direct function of $\sigma^2_{x,k}$ which is independent of M, and (6), in its simple form, can be used to analyze PS-CDMA systems.

B. Power-independent analysis

The functions in (6) and (7) depend on the receive power P_k of user k. As a result, the complete variance evolution involves analyzing a K-dimension problem. In this section,

[1] The results can be straightforwardly extended to an asynchronous channel.

[2] PSAD node here is identical to the variable node of degree M in an LDPC code

we show that under some mild assumptions we can translate the multi-dimensional analysis into an equivalent one-dimensional problem as in [5].

In a large-scale system, imposing the *Lindeberg condition* on asymptotic negligibility on the powers of the users, i.e., $\frac{P_k}{\sum_{j=1,j\neq k}^{K} P_j} \stackrel{K\to\infty}{\longrightarrow} 0, \forall k$, the limiting variance of the PSAD output $x_{k,m}$ becomes

$$\sigma^2(x_{k,m}) \implies \frac{M}{M-1}\left(\frac{1}{N}\sum_{j=1}^{K} P_j\sigma_{d,j}^2+\sigma^2\right), \quad (8)$$

which is constant for all users. Equation (8) provides an effective common variance evolution function in the form of

$$\overline{\sigma}_x^2 \triangleq \frac{\sigma^2(x_{k,m})}{\overline{P}} = \frac{M}{M-1}\left(\frac{K}{N}\overline{\sigma}_d^2+\frac{1}{2\overline{\gamma}}\right), \quad (9)$$

where $\overline{P}=\sum_{j=1}^{K} P_j/K$ is the average power of co-channel users, $\overline{\gamma}=\frac{\overline{P}}{2\sigma^2}$, $\overline{\sigma}_x^2$ is the variance of $x_{k,m}$ normalized w.r.t. $\overline{P}$, and $\overline{\sigma}_d^2$ is the effective symbol variance, defined by

$$\overline{\sigma}_d^2 = \sum_{j=1}^{K}\frac{P_j}{K\overline{P}}\sigma_{d,j}^2 = \sum_{j=1}^{K}\frac{P_j}{K\overline{P}} g\left(\frac{\overline{\sigma}_x^2\overline{P}}{P_j}\right) = f(\overline{\sigma}_x^2). \quad (10)$$

The effective variance evolutions in (9) and (10) allow us focus on the much simpler relationship between $\overline{\sigma}_d^2$ and $\overline{\sigma}_x^2$, which are independent of the specific user power profile $\{P_j\}_{j=1}^K$. To highlight the partitioned-spreading scheme and the variance evolution over stages, we put the effective functions as

$$\begin{aligned}\overline{\sigma}_{x,i}^2(M) &= \frac{M}{M-1}\left(\frac{K}{N}\overline{\sigma}_{d,i-1}^2(M)+\frac{1}{2\overline{\gamma}}\right) && (11)\\ \overline{\sigma}_{d,i}^2(M) &= f\left(\overline{\sigma}_{x,i}^2(M)\right), && (12)\end{aligned}$$

with i being the iteration stage of decoding.

Lemma 1: The functions $\overline{\sigma}_x^2$ and $\overline{\sigma}_d^2$ in (11) and (12) have at least one fix point.

Proof: In (11), $\overline{\sigma}_x^2$ from (9) is linear w.r.t. $\overline{\sigma}_d^2$, hence a monotonic function on $\overline{\sigma}_d^2$. It can be shown that $g(b)$ is a monotoic function of b using the results in [6]). As a result, $f(\overline{\sigma}_x^2)$ in (12) is also monotonic on $\overline{\sigma}_x^2$.

Consider the boundary values for both functions when $\overline{\sigma}_d^2 = 0$ and $\overline{\sigma}_d^2 = 1$. For (11), the two coordinate-pairs $(\overline{\sigma}_d^2, \overline{\sigma}_x^2)$ are $\left(1, \frac{M}{M-1}(\frac{K}{N}+\frac{1}{2\overline{\gamma}}\right)$ and $\left(0, \frac{M}{M-1}\frac{1}{2\overline{\gamma}}\right)$, while those for (12) are $(1,\infty)$ and $(0,0)$, respectively. Combining the monotonicity and boundary coordinates, it is evident that the two variance evolution functions intersect at least once.

Q. E. D.

Lemma 2: For a given number of iteration stages $i \in N^+$, $\overline{\sigma}_{x,i}^2(M) < \overline{\sigma}_{x,i}^2(M')$ if $M > M' \geq 2$.

Proof: The initial values on symbol variance $\overline{\sigma}_{x,0}^2(M) = \overline{\sigma}_{x,0}^2(M') = 1$. At the first iteration, the following inequality

$$\begin{aligned}\overline{\sigma}_{x,1}^2(M) &= \frac{M}{M-1}\left(\frac{K}{N}\overline{\sigma}_{d,0}^2(M)+\frac{1}{2\overline{\gamma}_k}\right)\\ &< \frac{M'}{M'-1}\left(\frac{K}{N}\overline{\sigma}_{d,0}^2(M')+\frac{1}{2\overline{\gamma}_k}\right) = \overline{\sigma}_{x,1}^2(M'),\end{aligned}$$

holds by noting that the elements in the set $\left\{\frac{M}{M-1} : M=2,3,\dots\right\}$ are monotonically decreasing. Consequently, $\overline{\sigma}_{d,1}^2(M) < \overline{\sigma}_{d,1}^2(M')$ using the monotonicity of function $f(b)$. The lemma is satisfied by repeating the above procedure until the i-th iteration.

Q. E. D.

Using Lemma 2, we can deduce the following proposition:

Proposition 1: The first fixed point $\overline{\sigma}_{x,\infty}^2(M) < \overline{\sigma}_{x,\infty}^2(M')$ if $M > M' \geq 2$.

Proof: The first fixed point is reached when the following equilibrium is achieved for the variance evolution functions

$$\begin{aligned}\overline{\sigma}_{x,\infty}^2(M) &= \frac{M}{M-1}\left(\frac{K}{N}\overline{\sigma}_{d,\infty}^2(M)+\frac{1}{2\overline{\gamma}_k}\right) && (13)\\ \overline{\sigma}_{d,\infty}^2(M) &= f\left(\overline{\sigma}_{x,\infty}^2(M)\right). && (14)\end{aligned}$$

From Lemma 2, we can infer that $\overline{\sigma}_{x,\infty}^2(M) \leq \overline{\sigma}_{x,\infty}^2(M')$ if $M > M'$. Now hypothesize that $\overline{\sigma}_{x,\infty}^2(M) = \overline{\sigma}_{x,\infty}^2(M')$, then from equation (14), we get

$$\begin{aligned}\overline{\sigma}_{x,\infty}^2(M) &= \frac{M}{M-1}\left(\frac{K}{N}\overline{\sigma}_{d,\infty}^2(M)+\frac{1}{2\overline{\gamma}_k}\right)\\ &< \frac{M'}{M'-1}\left(\frac{K}{N}\overline{\sigma}_{d,\infty}^2(M')+\frac{1}{2\overline{\gamma}_k}\right)\\ &= \overline{\sigma}_{x,\infty}^2(M'), && (15)\end{aligned}$$

which contradicts the hypothesis. Therefore, the hypothesis is false, and strict inequality in the proposition holds.

Q. E. D.

We emphasize that the results shown in this section are derived under the stipulation that an arbitrary user power profile $\{P_j\}_{j=1}^K$ is present in the channel, obeying only (8).

Figure 2 shows the VE functions for a PS-CDMA system with $K = 160$ users, spreading gain $N = 80$, and operating SNR $\overline{\gamma} = 6$ dB. The dashline represents equation (12), and the solid lines are function (11) for $M = 2, 4, 8$ and 80, respectively. The circles indicate the first fix point of the iterative process which decreases as M increases.

C. FEC coded partition-spreading CDMA

Up to this stage, we limit our discussion to the uncoded CDMA systems. Recent research [7], [8], [4] has shown the

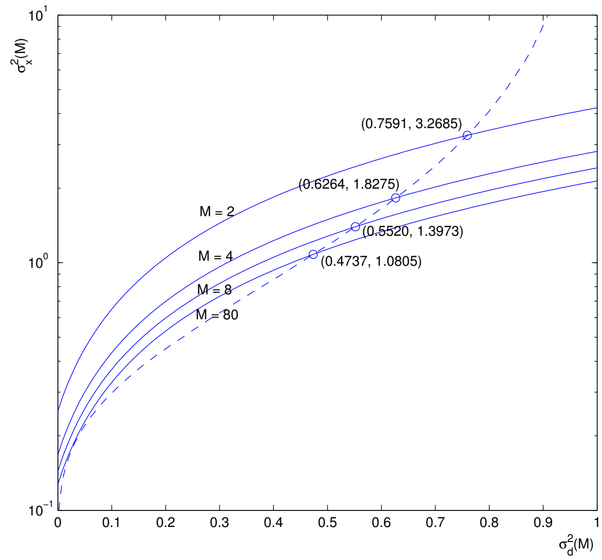

Fig. 2. Variance evolution curves for PS-CDMA systems with $K = 160, N = 80$, and average SNR $\overline{\gamma} = 6$ dB.

multiple-access channels that incorporate error control codes can achieve a performance superior to that of their uncoded counterparts. Hence it is worthwhile to investigate if PS-CDMA is preferable in the presence of FEC codes.

We denote the iteration between PSAD and MND in Figure 1 as the *multiple-access* (MA) stage, and that between multiuser-detection and FEC decoding as the *turbo* iteration. As shown in the previous section, at the first turbo iteration the multiuser detector of PS-CDMA with M partitions generates signals with higher SNR than systems with M' partitions ($M > M'$). The more reliable input in turn results in a superior output for the FEC decoder, which is then the input to the multiple-access stage in the next turbo iteration. Taking this initial condition, apply the monotonic arguments for variance evolution functions as in Section III-B, it can be shown that PS-CDMA with increasing M is also superior for later turbo iterations.

D. Variance evolution examples

- $M = N$**, IDMA** A special case of PS-CDMA scheme has been proposed by Li et. al. [10], [11], [12], [13] for $M = N$, when the number of partitions equals the spreading gain, i.e., IDMA. In IDMA, spreading is not necessary, and the multiple-access channel is highly interference-limited. The bandwidth expansion is used exclusively as virtual rate-$1/N$ repetition code. It was shown that IDMA can achieve a much better performance than traditional CDMA via BER simulations.
- $M = 1$**, CDMA** When $M = 1$, PS-CDMA reduces to the conventional CDMA system. The iterative decoding procedure in Figure 1 is not applicable, since PSAD faces a symbol without repetition, which corresponds to a trivial extrinsic output. Instead, the PSAD can feedback the *a posteriori* LLR, i.e., the incoming message itself. This is the so called *probabilistic data association* (PDA), a technique that has recently been used in multiuser detection [15].

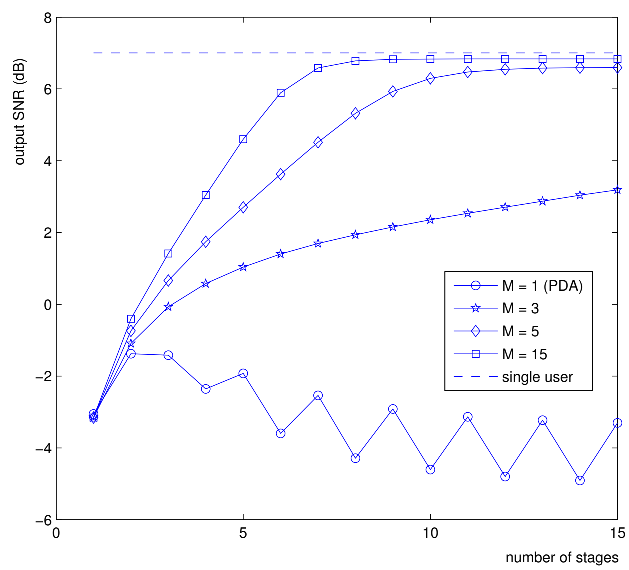

Fig. 3. Signal-to-noise ratio curves of PS-CDMA detection outputs with $M = 1, 3, 5, 15$, respectively. The number of co-channel users is $K = 15$, spreading gain $N = 15$, operating $\frac{E_b}{N_0} = 7$ dB.

Due to the APP feedback, correlation builds up over decoding iterations which causes a performance degeneration for PDA. Figure 3 shows the output SNR $\gamma_{\mathrm{APP}}(M) = 1/\sigma^2_{\mathrm{APP}}(M)$ of a fully loaded ($K = 15, N = 15$) CDMA systems with $M = 1, 3, 5$ and 15 partitions. The operating SNR is $\frac{E_b}{N_0} = 7$ dB. It is shown that SNR of a PS-CDMA system improves at every stage as M increases, which verifies the monotonicity of its output variance. The evolution of system performance of the PDA detector for $M = 1$ demonstrates a ping-pong effect, while for systems with $M > 1$ the SNR increases smoothly over stages.

E. Remark

PS-CDMA can be viewed as a technique that flexibly allocates the system dimension between *spreading* and the intrinsic *repetition code* to maximize the channel throughput. Based on the variance evolution analysis presented in Section III-B, we advocate the argument that the system dimension should be devoted to the repetition code, the simplest but an efficient error control coding technique in the context of multiple-access signaling. This observation is compatible with the argument that a conventional CDMA system can achieve the channel capacity by employing very low rate error control codes [16], [17].

On the other hand, Figure 3 shows that the improvement is significant by increasing the number of partitions from $M = 1$ to $M = 3$, while it becomes marginal from $M = 5$ to $M = 15$ (IDMA). Therefore, partitioning a CDMA symbol into a few number of portions is enough to fulfill a data rate which is close to that of an IDMA. Furthermore, the use of small number of partitions imposes less stringent requirement on interleaving as well as dynamic range for operating LLR $x_{k,m}$ in (5). Based on these arguments, we recommend a PS-CDMA system with a moderate number of partitions instead of IDMA for practical implementation.

IV. Intrinsic channel throughput of PS-CDMA

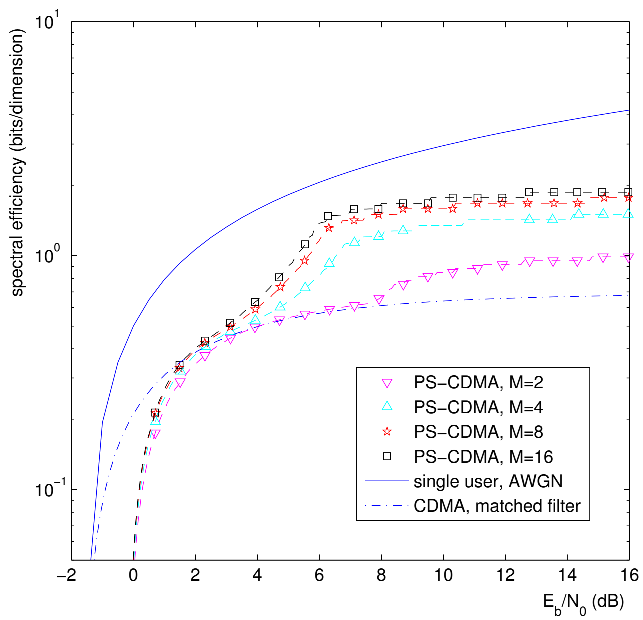

Fig. 4. Spectral efficiency for equal-power PS-CDMA systems with $M = 2, 4, 8$ and 16, respectively.

We propose to consider PS-CDMA as a non-linear separation filter that links the binary input $d_k \in \{-1, +1\}$ and the Gaussian output $\sum_{m=1}^{M} x_{k,m}$. Rather than a detection device, it is a signal-separator which separates transmission channels for all users. Given system load $\alpha = \frac{K}{N}$ and signal-to-noise ratio γ, the layered-channel for user k is limited by $\sigma^2_{\text{APP}}(\alpha, M)$ that can be calculated through variance evolution. Accordingly, the capacity of the underlying Gaussian channel for user k can be obtained by

$$C_k(\alpha, M) = C_{\text{bpsk}}\left(\frac{P_k}{2\sigma^2_{\text{APP}}(\alpha, M)}\right) \tag{16}$$

$$C_{\text{bpsk}}(\gamma) = 1 - \sum_{b\in(-1,+1)} \frac{1}{2}\int_{-\infty}^{\infty} \frac{e^{-z^2/2}}{\sqrt{2\pi}} \log \left[\sum_{a\in(-1,+1)} e^{-2\gamma(b-a)^2} e^{-\sqrt{2\gamma}(b-a)z}\right] dz. \tag{17}$$

The spectral efficiency of the PS-CDMA system is defined as the maximum throughput that is normalized w.r.t. the dimension N, given by,

$$C(M) = \max_{\alpha} \frac{1}{N}\sum_{k=1}^{K} C_{\text{bpsk}}\left(\frac{P_k}{2\sigma^2_{\text{APP}}(\alpha, M)}\right), \quad \text{with} \quad \frac{E_b}{N_0} = \frac{P_k}{2C_k(\alpha, M)\sigma^2} \quad \text{fixed} \tag{18}$$

Figure 4 shows the spectral efficiency for equal-power PS-CDMA systems with partition numbers ranging from $M = 2$ to $M = 16$. It can be seen that PS-CDMA with more than four partitions enables a much higher channel throughput than that of the CDMA with Matched filter when $E_b/N_0 \geq 3$ dB.

In order to approach the spectral efficiency in (18), the SNR at the output of the multistage demodulator may be lower than that of the single-user channel. The left-hand side of Figure 5 shows the per-user SNR $\gamma_{\text{pu}} = P_k/2\sigma^2_{\text{APP}}(\alpha, M)$ for the output of the PS-CDMA. It can be seen that for $\gamma < 6$ dB, there is a non-trivial gap between γ_{pu} and single-user channel SNR γ for PS-CDMA with $M \geq 4$. The right-hand side of Figure 5 are the capacity for each layered channel, which indicates the appropriate coding rate to achieve the maximum channel throughput. Though varying in values, the optimal coding rates for all PS-CDMA systems approach one as the SNR increases. Therefore, most of the bandwidth expansion goes to the repetition coding/spreading, which addresses the multiple-access interference, while high-rate FEC code is used to remove the residual error from the resulting single-user channel. This justifies the low-complexity two-stage LDPC encoded PS-CDMA system proposed by Schlegel et. al. in [3].

V. Conclusions

In this paper, we investigate the performance of general PS-CDMA systems which partition the conventional CDMA symbol into a number of sections and transmit them in random order. PS-CDMA is flexible in allocating the system dimension into spreading and repetition coding, and superior performance of the system can be realized by employing a multi-stage demodulator. The dynamics of PS-CDMA are shown to improve as the number of partitions M increases, for arbitrary user-power profiles, while the benefit diminishes as M becomes larger. The spectral efficiency of the PS-CDMA system is shown to be much higher than that for conventional CDMA, and can be achieved by using the combination of low rate repetition coding and high rate FEC codes.

Acknowledgment

Z. Shi and M. C. Reed are with National ICT Australia and affiliated with the Australian National University. National ICT Australia is funded through the Australian Government's *backing Australia's Ability* initiative and in part through the Australia Research Council.

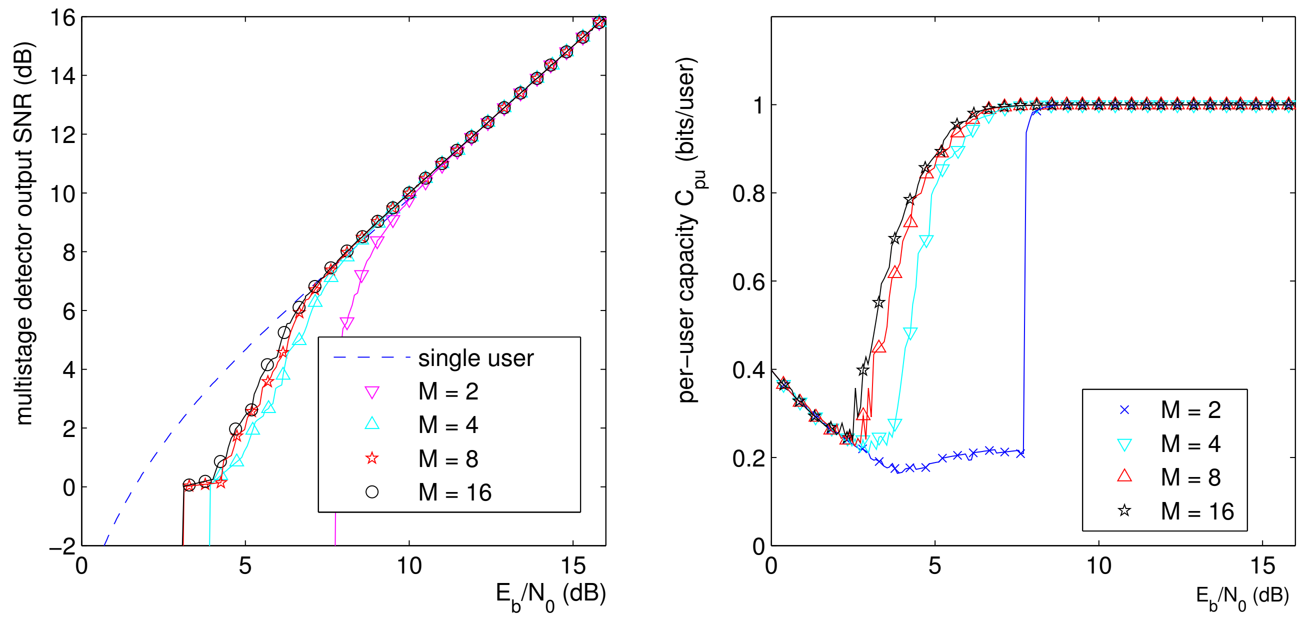

Fig. 5. Per-user SNR and capacity to maximize PS-CDMA throughput.

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